Minicourse: Non-equilibrium steady states of classical chains of oscillators / rotors

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Tuesday:

1. **Introduction**. Chains of oscillators / rotors. Langevin heat baths. NESS.
2. **Properties of the NESS.** Known results. Temperature profiles. Heat conduction. [2, 6, 11, 17, 19] for rotors, [1–3, 12, 18, 19] for oscillators.
3. **Interaction versus pinning.** Polynomial potentials. Frequency of an isolated oscillator. When the interactions dominate [4, 8–10, 22]. When the pinning dominates [14]. Rotators as a limit of infinite pinning [5].
4. **Irreducibility and technicalities.** Hörmander's bracket condition [16, 21]. Equivalence with equilibrium. Irreducibility [5, 13, 20].
5. **Strongly interacting oscillators.** Exponential convergence to a NESS. Scaling at high energy. [4, 8–10, 22]
6. **Subgeometric ergodicity criterion.** φ-Lyapunov functions [7]. Examples. Stretched exponential convergence rates.

Wednesday:

7. **Three rotors.** Asymptotic decoupling. Averaging rapid oscillations. Lyapunov function. Stretched exponential convergence to the NESS [5], method from [15].
8. **Three strongly pinned oscillators.** From rotors to strongly pinned oscillators. [15]
9. **Four rotors.** From 3 to 4 rotors. Resonances. Decoupled dynamics approximation. Lyapunov function. Stretched exponential convergence to the NESS.
10. **Open problems.** A few open problems. Arbitrary potentials. Hybrid chains. More realistic heat baths. Networks.

References

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